HOOFDSTUK 7: Differentiaalrekening

7.1 De afgeleide van gebroken functies

Opgave 1:

a. $p(x) = x^2(3x-7) = 3x^3 - 7x^2$

$p'(x) = 9x^2 - 14x$

$f'(x) = 2x$

$g'(x) = 3$

b. $f'(x) \cdot g'(x) = 2x \cdot 3 = 6x \neq p'(x)$

c. $f'(x) \cdot g(x) + f(x) \cdot g'(x) = 2x \cdot (3x-7) + x^2 \cdot 3 = 6x^2 - 14x + 3x = 9x^2 - 14x = p'(x)$

Opgave 2:

a. $f'(x) = -6x(2+7x) + (2-3x^2) \cdot 7$

b. $g'(x) = 2(2x-5) + (2x-5) \cdot 2$

c. $h'(x) = (2x-3)(x^3 + x^2 + x) + (x^2 - 3x)(3x^2 + 2x + 1)$

d. $j'(x) = 6x(3x^2 - 4) + (3x^2 - 4) \cdot 6x$

Opgave 3:

a. $p' = [f \cdot g]' \cdot h + [f \cdot g] \cdot h' = (f' \cdot g + f \cdot g') \cdot h + f \cdot g \cdot h' =$

$f' \cdot g \cdot h + f \cdot g' \cdot h + f \cdot g \cdot h'$

b. $p' = f' \cdot g \cdot h \cdot j + f \cdot g' \cdot h \cdot j + f \cdot g \cdot h' \cdot j + f \cdot g \cdot h \cdot j'$

Opgave 4:

$q(x) = \dfrac{t(x)}{n(x)}$

Kruiselings vermenigvuldigen geeft $q(x) \cdot n(x) = t(x)$

Differentiëren van het linker- en rechterlid geeft

$q'(x) \cdot n(x) + q(x) \cdot n'(x) = t'(x)$

$q'(x) \cdot n(x) = t'(x) - q(x) \cdot n'(x)$

$q'(x) = \dfrac{t'(x) - q(x) \cdot n'(x)}{n(x)}$

Substitutie van $q(x) = \dfrac{t(x)}{n(x)}$ geeft $q'(x) = \dfrac{t'(x) - \dfrac{t(x)}{n(x)} \cdot n'(x)}{n(x)}$

Teller en noemer vermenigvuldigen met $n(x)$ geeft $q'(x) = \dfrac{n(x) \cdot t'(x) - t(x) \cdot n'(x)}{(n(x))^2}$

Opgave 5:

a. $f'(x) = \dfrac{(x+5) - (x-2)}{(x+5)^2} = \dfrac{x+5-x+2}{(x+5)^2} = \dfrac{7}{(x+5)^2}$

b. $g'(x) = \dfrac{(-x+6) \cdot 3 - (3x+2) \cdot -1}{(-x+6)^2} = \dfrac{-3x+18+3x+2}{(-x+6)^2} = \dfrac{20}{(-x+6)^2}$

c. $h'(x)=\frac{(2x-1)\cdot 0-2\cdot 2}{(2x-1)^2}=\frac{-4}{(2x-1)^2}$

d. $j(x)=\frac{6x-9}{3}=2x-3$

$j'(x)=2$

Opgave 6:

a. $f'(x)=\frac{(2x^2+1)\cdot 3x^2-x^3\cdot 4x}{(2x^2+1)^2}=\frac{6x^4+3x^2-4x^4}{(2x^2+1)^2}=\frac{2x^4+3x^2}{(2x^2+1)^2}$

b. $g'(x)=\frac{(3-x^2)\cdot 1-(x-2)\cdot -2x}{(3-x^2)^2}=\frac{3-x^2+2x^2-4x}{(3-x^2)^2}=\frac{x^2-4x+3}{(3-x^2)^2}$

c. $h'(x)=\frac{(x-2)\cdot -2x-(3-x^2)\cdot 1}{(x-2)^2}+3x^2=\frac{-2x^2+4x-3+x^2}{(x-2)^2}+3x^2=\frac{-x^2+4x-3}{(x-2)^2}+3x^2$

d. $j'(x)=1-\frac{(x+4)\cdot 0-2\cdot 1}{(x+4)^2}=1+\frac{2}{(x+4)^2}$

Opgave 7:

a. $\frac{x^2-4}{2x+5}=0$

$x^2-4=0$

$x^2=4$

$x=2 \quad \vee \quad x=-2$

$f'(x)=\frac{(2x+5)\cdot 2x-(x^2-4)\cdot 2}{(2x+5)^2}=\frac{4x^2+10x-2x^2+8}{(2x+5)^2}=\frac{2x^2+10x+8}{(2x+5)^2}$

in $(-2,0)$	in $(2,0)$
$rc=f'(-2)=-4$	$rc=f'(2)=\frac{4}{9}$
$y=-4x+b$ door $(-2,0)$	$y=\frac{4}{9}x+b$ door $(2,0)$
$0=8+b$	$0=\frac{8}{9}+b$
$b=-8$	$b=-\frac{8}{9}$
$y=-4x-8$	$y=\frac{4}{9}x-\frac{8}{9}$

b. $y_C=f(0)=-\frac{4}{5}$

$rc=f'(0)=\frac{8}{25}$

$y=\frac{8}{25}x+b$ door $(0,-\frac{4}{5})$

$-\frac{4}{5}=0+b$

$b=-\frac{4}{5}$

$y=\frac{8}{25}x-\frac{4}{5}$

c. $f'(x)=\frac{2x^2+10x+8}{(2x+5)^2}=0$

$2x^2+10x+8=0$

$x^2+5x+4=0$

$(x+1)(x+4)=0$

$x = -1 \quad \vee \quad x = -4$

$y = -1 \qquad\quad y = -4$

dus $(-1,-1)$ en $(-4,-4)$

Opgave 8:

$y_A = \frac{5}{4}$

$\frac{2x-5}{x^2-4} = 0$

$2x - 5 = 0$

$2x = 5$

$x_B = 2\frac{1}{2}$

$f'(x) = \frac{(x^2-4)\cdot 2 - (2x-5)\cdot 2x}{(x^2-4)^2} = \frac{2x^2 - 8 - 4x^2 + 10x}{(x^2-4)^2} = \frac{-2x^2 + 10x - 8}{(x^2-4)^2}$

$rc = f'(0) = -\frac{1}{2}$

$y = -\frac{1}{2}x + b$ door $(0, \frac{5}{4})$

$b = \frac{5}{4}$

$y = -\frac{1}{2}x + \frac{5}{4}$

$-\frac{1}{2}x + \frac{5}{4} = 0$

$-\frac{1}{2}x = -\frac{5}{4}$

$x = 2\frac{1}{2}$ dus de bewering is waar

7.2 De afgeleide van machtfuncties

Opgave 9:

a. $\dfrac{1}{x^3} = x^{-3}$

$\dfrac{5}{x^4} = 5x^{-4}$

$\dfrac{1}{3x^2} = \tfrac{1}{3}x^{-2}$

b. $x^{-4} = \dfrac{1}{x^4}$

$3x^{-2} = \dfrac{3}{x^2}$

$\tfrac{1}{7}x^{-6} = \dfrac{1}{7x^6}$

Opgave 10:

a. $\dfrac{x^3 + 5x^2}{x} = x^2 + 5x$

$\dfrac{4x^2 + 7x}{x^3} = \dfrac{4x^2}{x^3} + \dfrac{7x}{x^3} = 4x^{-1} + 7x^{-2}$

$\dfrac{2x^5 + 5x^2}{3x^4} = \dfrac{2x^5}{3x^4} + \dfrac{5x^2}{3x^4} = \tfrac{2}{3}x + \tfrac{5}{3}x^{-2}$

b. $\dfrac{1}{2x} + \dfrac{2}{x^2} = \dfrac{x}{2x^2} + \dfrac{4}{2x^2} = \dfrac{x+4}{2x^2}$

$\tfrac{1}{2}x + \dfrac{3}{x^2} = \dfrac{x^3}{2x^2} + \dfrac{6}{2x^2} = \dfrac{x^3+6}{2x^2}$

$\tfrac{2}{3}x^2 - \dfrac{3}{4x} = \dfrac{8x^3}{12x} - \dfrac{9}{12x} = \dfrac{8x^3-9}{12x}$

Opgave 11:

a. $\left[\dfrac{1}{x^2}\right]' = \dfrac{x^2 \cdot 0 - 1 \cdot 2x}{x^4} = -\dfrac{2x}{x^4} = -\dfrac{2}{x^3}$

b. $\left[x^{-2}\right]' = \left[\dfrac{1}{x^2}\right]' = -\dfrac{2}{x^3} = -2x^{-3}$

c. $\left[x^{-5}\right]' = \left[\dfrac{1}{x^5}\right]' = \dfrac{x^5 \cdot 0 - 1 \cdot 5x^4}{x^{10}} = -\dfrac{5x^4}{x^{10}} = -5x^{-6}$

Opgave 12:

a. omdat de noemer uit één term bestaat kun je uitdelen waardoor je losse termen krijgt die je stuk voor stuk kunt differentiëren.

b. $g(x)$ en $h(x)$

Opgave 13:

a. $f(x) = \frac{1}{x^6} = x^{-6}$

$f'(x) = -6x^{-7} = -\frac{6}{x^7}$

b. $g(x) = 5 - \frac{3}{x^2} = 5 - 3x^{-2}$

$g'(x) = 6x^{-3} = \frac{6}{x^3}$

c. $h(x) = ax^4 - \frac{b}{x^4} = ax^4 - bx^{-4}$

$h'(x) = 4ax^3 + 4bx^{-5} = 4ax^3 + \frac{4b}{x^5}$

Opgave 14:

a. $f'(x) = \frac{3x^2 \cdot 2 - (2x-1) \cdot 6x}{9x^4} = \frac{6x^2 - 12x^2 + 6x}{9x^4} = \frac{-6x^2 + 6x}{9x^4} = \frac{-2x+2}{3x^3}$

b. $g'(x) = \frac{(2x-1) \cdot 6x - 3x^2 \cdot 2}{(2x-1)^2} = \frac{12x^2 - 6x - 6x^2}{(2x-1)^2} = \frac{6x^2 - 6x}{(2x-1)^2}$

c. $h'(x) = \frac{x^3 \cdot 18x^5 - (3x^6 - 3) \cdot 3x^2}{x^6} = \frac{18x^8 - 9x^8 + 9x^2}{x^6} = \frac{9x^8 + 9x^2}{x^6} = \frac{9x^6 + 9}{x^4}$

Opgave 15:

a. $y_A = \frac{2}{3}$

$f'(x) = \frac{(x^2-1) \cdot 1 - x \cdot 2x}{(x^2-1)^2} = \frac{x^2 - 1 - 2x^2}{(x^2-1)^2} = \frac{-x^2 - 1}{(x^2-1)^2}$

$rc = f'(2) = -\frac{5}{9}$

$y = -\frac{5}{9}x + b$ door $(2, \frac{2}{3})$

$\frac{2}{3} = -\frac{10}{9} + b$

$b = 1\frac{7}{9}$

$y = -\frac{5}{9}x + 1\frac{7}{9}$

b. $y_B = 1\frac{1}{2}$

$g'(x) = \frac{x \cdot 2x - (x^2-1) \cdot 1}{x^2} = \frac{2x^2 - x^2 + 1}{x^2} = \frac{x^2+1}{x^2}$

$rc = g'(2) = \frac{5}{4}$

$y = \frac{5}{4}x + b$ door $(2, 1\frac{1}{2})$

$1\frac{1}{2} = 2\frac{1}{2} + b$

$b = -1$

$y = \frac{5}{4}x - 1$

c. $\frac{x^2 - 1}{x} = 0$

$x^2 - 1 = 0$

$x^2 = 1$

$x = 1 \quad \vee \quad x = -1$

$rc = g'(1) = 2$	$rc = g'(-1) = 2$
$y = 2x + b$ door $(1,0)$	$y = 2x + b$ door $(-1,0)$
$0 = 2 + b$	$0 = -2 + b$
$b = -2$	$b = 2$
$y = 2x - 2$	$y = 2x + 2$

Opgave 16:

a. $\dfrac{x^2}{\sqrt{x}} = \dfrac{x^2}{x^{\frac{1}{2}}} = x^{1\frac{1}{2}}$

$\sqrt[3]{x^2} = x^{\frac{2}{3}}$

$\dfrac{x^2 \cdot \sqrt{x}}{x^4} = \dfrac{x^2 \cdot x^{\frac{1}{2}}}{x^4} = \dfrac{x^{2\frac{1}{2}}}{x^4} = x^{-1\frac{1}{2}}$

b. $x^{\frac{1}{5}} = \sqrt[5]{x}$

$x^{2\frac{1}{2}} = x^2 \cdot x^{\frac{1}{2}} = x^2 \cdot \sqrt{x}$

$x^{-1\frac{1}{3}} = \dfrac{1}{x^{1\frac{1}{3}}} = \dfrac{1}{x \cdot x^{\frac{1}{3}}} = \dfrac{1}{x \cdot \sqrt[3]{x}}$

Opgave 17:

a. $[x]' = \left[x^{\frac{1}{2}} \cdot x^{\frac{1}{2}}\right]' = \left[x^{\frac{1}{2}}\right]' \cdot x^{\frac{1}{2}} + x^{\frac{1}{2}} \cdot \left[x^{\frac{1}{2}}\right]' = 2 \cdot x^{\frac{1}{2}} \cdot \left[x^{\frac{1}{2}}\right]' = 1$

b. $\left[x^{\frac{1}{2}}\right]' = \dfrac{1}{2x^{\frac{1}{2}}} = \frac{1}{2}x^{-\frac{1}{2}}$

Opgave 18:

a. $f(x) = x + \sqrt{x} = x + x^{\frac{1}{2}}$

$f'(x) = 1 + \frac{1}{2}x^{-\frac{1}{2}} = 1 + \dfrac{1}{2x^{\frac{1}{2}}} = 1 + \dfrac{1}{2 \cdot \sqrt{x}}$

b. $g(x) = x \cdot \sqrt[3]{x} = x^1 \cdot x^{\frac{1}{3}} = x^{1\frac{1}{3}}$

$g'(x) = 1\frac{1}{3}x^{\frac{1}{3}} = 1\frac{1}{3} \cdot \sqrt[3]{x}$

c. $h(x) = \dfrac{1}{\sqrt{x}} = \dfrac{1}{x^{\frac{1}{2}}} = x^{-\frac{1}{2}}$

$h'(x) = -\frac{1}{2}x^{-1\frac{1}{2}} = \dfrac{-1}{2x^{1\frac{1}{2}}} = \dfrac{-1}{2x \cdot \sqrt{x}}$

d. $j(x) = x^3 \cdot \sqrt[5]{x^3} = x^3 \cdot x^{\frac{3}{5}} = x^{3\frac{3}{5}}$

$j'(x) = 3\frac{3}{5}x^{2\frac{3}{5}} = 3\frac{3}{5}x^2 \cdot x^{\frac{3}{5}} = 3\frac{3}{5}x^2 \cdot \sqrt[5]{x^3}$

e. $k(x) = x^2 \cdot \sqrt[4]{x} = x^2 \cdot x^{\frac{1}{4}} = x^{2\frac{1}{4}}$

$k'(x) = 2\frac{1}{4}x^{1\frac{1}{4}} = 2\frac{1}{4}x^1 \cdot x^{\frac{1}{4}} = 2\frac{1}{4}x\sqrt[4]{x}$

f. $l(x) = (x^2 + 1)(1 + \sqrt{x}) = x^2 + x^2 \cdot \sqrt{x} + 1 + \sqrt{x} = x^2 + x^{2\frac{1}{2}} + 1 + x^{\frac{1}{2}}$

$$l'(x) = 2x + 2\tfrac{1}{2}x^{1\frac{1}{3}} + \tfrac{1}{2}x^{-\frac{1}{2}} = 2x + 2\tfrac{1}{2}x \cdot \sqrt{x} + \frac{1}{2x^{\frac{1}{2}}} = 2x + 2\tfrac{1}{2}x \cdot \sqrt{x} + \frac{1}{2\sqrt{x}}$$

Opgave 19:

a. $f(x) = (x \cdot \sqrt{x} - 3)^2 = x^3 - 6x \cdot \sqrt{x} + 9 = x^3 - 6x^{1\frac{1}{2}} + 9$

$f'(x) = 3x^2 - 9x^{\frac{1}{2}} = 3x^2 - 9\sqrt{x}$

b. $$g'(x) = \frac{(x+1) \cdot 1\frac{1}{2}\sqrt{x} - x\sqrt{x} \cdot 1}{(x+1)^2} = \frac{1\frac{1}{2}x\sqrt{x} + 1\frac{1}{2}\sqrt{x} - x\sqrt{x}}{(x+1)^2} = \frac{\frac{1}{2}x\sqrt{x} + 1\frac{1}{2}\sqrt{x}}{(x+1)^2}$$

c. $$h'(x) = \frac{(x^2+2) \cdot \frac{1}{\sqrt{x}} - 2\sqrt{x} \cdot 2x}{(x^2+2)^2} = \frac{\frac{x^2}{\sqrt{x}} + \frac{2}{\sqrt{x}} - 4x\sqrt{x}}{(x^2+2)^2} = \frac{x\sqrt{x} + \frac{2}{\sqrt{x}} - 4x\sqrt{x}}{(x^2+2)^2} =$$

$$\frac{\frac{2}{\sqrt{x}} - 3x\sqrt{x}}{(x^2+2)^2} = \frac{\frac{2}{\sqrt{x}} - \frac{3x^2}{\sqrt{x}}}{(x^2+2)^2} = \frac{\frac{2-3x^2}{\sqrt{x}}}{(x^2+2)^2} = \frac{2-3x^2}{\sqrt{x} \cdot (x^2+2)^2}$$

Opgave 20:

a. $$f(x) = \frac{x+1}{\sqrt{x}} = \frac{x}{\sqrt{x}} + \frac{1}{\sqrt{x}} = x^{\frac{1}{2}} + x^{-\frac{1}{2}}$$

$$f'(x) = \tfrac{1}{2}x^{-\frac{1}{2}} - \tfrac{1}{2}x^{-1\frac{1}{2}} = \frac{1}{2\sqrt{x}} - \frac{1}{2x\sqrt{x}} = \frac{x}{2x\sqrt{x}} - \frac{1}{2x\sqrt{x}} = \frac{x-1}{2x\sqrt{x}}$$

b. $$g(x) = \frac{x+1}{x\sqrt{x}} = \frac{x}{x\sqrt{x}} + \frac{1}{x\sqrt{x}} = x^{-\frac{1}{2}} + x^{-1\frac{1}{2}}$$

$$g'(x) = -\tfrac{1}{2}x^{-1\frac{1}{2}} - 1\tfrac{1}{2}x^{-2\frac{1}{2}} = \frac{-1}{2x^{1\frac{1}{2}}} - \frac{3}{2x^{2\frac{1}{2}}} = \frac{-1}{2x\sqrt{x}} - \frac{3}{2x^2\sqrt{x}} = \frac{-x}{2x^2\sqrt{x}} - \frac{3}{2x^2\sqrt{x}} = \frac{-x-3}{2x^2\sqrt{x}}$$

c. $$h(x) = \frac{x^2+2}{2\sqrt{x}} = \frac{x^2}{2\sqrt{x}} + \frac{2}{2\sqrt{x}} = \tfrac{1}{2}x^{1\frac{1}{2}} + x^{-\frac{1}{2}}$$

$$h'(x) = \tfrac{3}{4}x^{\frac{1}{2}} - \tfrac{1}{2}x^{-1\frac{1}{2}} = \tfrac{3}{4}\sqrt{x} - \frac{1}{2x\sqrt{x}} = \frac{3x^2}{4x\sqrt{x}} - \frac{2}{4x\sqrt{x}} = \frac{3x^2-2}{4x\sqrt{x}}$$

Opgave 21:

$y_A = f(\tfrac{1}{8}) = \tfrac{1}{4} \qquad y_B = f(8) = 4$

$f(x) = \sqrt[3]{x^2} = x^{\frac{2}{3}}$

$$f'(x) = \tfrac{2}{3}x^{-\frac{1}{3}} = \frac{2}{3 \cdot \sqrt[2]{x}}$$

$f'(\tfrac{1}{8}) = \tfrac{4}{3}$ $\qquad$ $f'(8) = \tfrac{1}{3}$

$y = \tfrac{4}{3}x + b$ door $(\tfrac{1}{8}, \tfrac{1}{4})$ $\qquad$ $y = \tfrac{1}{3}x + b$ door $(8,4)$

$\tfrac{1}{4} = \tfrac{1}{6} + b$ $\qquad$ $4 = \tfrac{8}{3} + b$

$b = \tfrac{1}{12}$ $\qquad$ $b = \tfrac{4}{3}$

$y = \tfrac{4}{3}x + \tfrac{1}{12}$ $\qquad$ $y = \tfrac{1}{3}x + \tfrac{4}{3}$

$\tfrac{4}{3}x + \tfrac{1}{12} = \tfrac{1}{3}x + \tfrac{4}{3}$

$x = 1\frac{1}{4}$
$y = 1\frac{3}{4}$
Dus $C = (1\frac{1}{4}, 1\frac{3}{4})$

Opgave 22:

a. $f(x) = x\sqrt{x} - 3x = x^{1\frac{1}{2}} - 3x$
$f'(x) = 1\frac{1}{2}x^{\frac{1}{2}} - 3 = 1\frac{1}{2}\sqrt{x} - 3$
$f'(0) = -3$
$y = -3x + b$ door $(0,0)$
$b = 0$
$y = -3x$

b. $f'(x) = 1\frac{1}{2}\sqrt{x} - 3 = 3$
$1\frac{1}{2}\sqrt{x} = 6$
$\sqrt{x} = 4$
$x = 16$
$y = f(16) = 16$
$y = 3x + b$ door $(16,16)$
$16 = 48 + b$
$-32 = b$
$y = 3x - 32$

Opgave 23:

$y_A = f(4) = 8$
$$f'(x) = \frac{(x+1)\cdot 7\frac{1}{2}\sqrt{x} - 5x\sqrt{x}\cdot 1}{(x+1)^2} = \frac{7\frac{1}{2}x\sqrt{x} + 7\frac{1}{2}\sqrt{x} - 5x\sqrt{x}}{(x+1)^2} = \frac{2\frac{1}{2}x\sqrt{x} + 7\frac{1}{2}\sqrt{x}}{(x+1)^2}$$
$f'(4) = 1\frac{2}{5}$
$y = 1\frac{2}{5}x + b$ door $(4,8)$
$8 = 5\frac{3}{5} + b$
$2\frac{2}{5} = b$
$y = 1\frac{2}{5}x + 2\frac{3}{5}$
$1\frac{2}{5}x + 2\frac{2}{5} = 0$
$1\frac{2}{5}x = -2\frac{2}{5}$
$x = -1\frac{5}{7}$
$A(4,8)$ en $B(-1\frac{5}{7}, 0)$
$Opp(\Delta ABC) = \frac{1}{2}\cdot 1\frac{5}{7}\cdot 8 = 6\frac{6}{7}$

Opgave 24:

a. $s'(t) = 15\sqrt{t}$
$s'(1) = 15\ {}^{m}\!/_{s}$

b. $v = 108\ {}^{km}\!/_{uur} = 30\ {}^{m}\!/_{s}$
$15\sqrt{t} = 30$

$\sqrt{t} = 2$

$t = 4$ sec

c. $s(9) = 270$

$s'(9) = 45 \, ^m\!/_s$

$\text{afstand} = 270 + 51 \cdot 45 = 2565 \text{ meter}$

7.3 De kettingregel

Opgave 25:

a. $f(x)=(x^2-5x)^2=x^4-10x^3+25x^2$

$f'(x)=4x^3-30x^2+50x$

b. $2(x^2-5x)\cdot\left[x^2-5x\right]'=2(x^2-5x)(2x-5)=(x^2-5x)(4x-10)=$

$=4x^3-10x^2-20x^2+50x=4x^3-30x^2+50x=f'(x)$

Opgave 26:

$y_2=y_3$

Opgave 27:

a. $f(x)=-2(2x+1)^4=-2u^4$ met $u=2x+1$ dus $u'=2$

$f'(x)=-8u^3\cdot u'=-8(2x+1)^3\cdot 2=-16(2x+1)^3$

b. $g(x)=\dfrac{1}{(3x-2)^2}=(3x-2)^{-2}=u^{-2}$ met $u=3x-2$ dus $u'=3$

$g'(x)=-2u^{-3}\cdot u'=-\dfrac{2}{u^3}\cdot u'=-\dfrac{2}{(3x-2)^3}\cdot 3=-\dfrac{6}{(3x-2)^3}$

c. $h(x)=\sqrt{2x^2+4x}=\sqrt{u}$ met $u=2x^2+4x$ dus $u'=4x+4$

$h'(x)=\dfrac{1}{2\sqrt{u}}\cdot u'=\dfrac{1}{2\sqrt{2x^2+4x}}\cdot(4x+4)=\dfrac{2x+2}{\sqrt{2x^2+4x}}$

d. $j(x)=\dfrac{1}{\sqrt{4x-1}}=(4x-1)^{-\frac{1}{2}}=u^{-\frac{1}{2}}$ met $u=4x-1$ dus $u'=4$

$j'(x)=-\tfrac{1}{2}u^{-1\frac{1}{2}}\cdot u'=\dfrac{-1}{2u\sqrt{u}}\cdot u'=\dfrac{-1}{2(4x-1)\sqrt{4x-1}}\cdot 4=\dfrac{-2}{(4x-1)\sqrt{4x-1}}$

e. $k(x)=(x^2+3)\sqrt{x^2+3}=(x^2+3)^{1\frac{1}{2}}=u^{1\frac{1}{2}}$ met $u=x^2+3$ dus $u'=2x$

$k'(x)=1\tfrac{1}{2}u^{\frac{1}{2}}\cdot u'=1\tfrac{1}{2}\sqrt{x^2+3}\cdot 2x=3x\sqrt{x^2+3}$

f. $l(x)=\dfrac{1}{\sqrt{x^2+2x+3}}=(x^2+2x+3)^{-\frac{1}{2}}=u^{-\frac{1}{2}}$ met $u=x^2+2x+3$ dus $u'=2x+2$

$l'(x)=-\tfrac{1}{2}u^{-1\frac{1}{2}}\cdot u'=\dfrac{-1}{2u\sqrt{u}}\cdot u'=\dfrac{-1}{2(x^2+2x+3)\sqrt{x^2+2x+3}}\cdot(2x+2)=$

$=\dfrac{-x-1}{(x^2+2x+3)\sqrt{x^2+2x+3}}$

Opgave 28:

a. $f(x)=4(x^3+7x-2)^2=4u^2$ met $u=x^3+7x-2$ dus $u'=3x^2+7$

$f'(x)=8u\cdot u'=8(x^3+7x-2)(3x^2+7)$

b. $g(x)=-\dfrac{6}{(x^2+3x)^3}=-\dfrac{6}{u^3}=-6u^{-3}$ met $u=x^2+3x$ dus $u'=2x+3$

$g'(x) = 18u^{-4} \cdot u' = \frac{18u'}{u^4} = \frac{18(2x+3)}{(x^2+3x)^4}$

c. $h(x) = \sqrt[3]{x^3+3x} = \sqrt[3]{u} = u^{\frac{1}{3}}$ met $u = x^3+3x$ dus $u' = 3x^2+3$

$h'(x) = \frac{1}{3}u^{-\frac{2}{3}} \cdot u' = \frac{1}{3u^{\frac{2}{3}}} \cdot u' = \frac{u'}{3 \cdot \sqrt[3]{u^2}} = \frac{3x^2+3}{3 \cdot \sqrt[3]{(x^3+3x)^2}} = \frac{x^2+1}{\sqrt[3]{(x^3+3x)^2}}$

d. $j(x) = \frac{1}{(4-x)\sqrt{4-x}} = \frac{1}{(4-x)^{1\frac{1}{2}}} = \frac{1}{u^{1\frac{1}{2}}} = u^{-1\frac{1}{2}}$ met $u = 4-x$ dus $u' = -1$

$j'(x) = -1\frac{1}{2}u^{-2\frac{1}{2}} \cdot u' = \frac{-3}{2u^{2\frac{1}{2}}} \cdot u' = \frac{-3u'}{2u^2\sqrt{u}} = \frac{3}{2(4-x)^2 \cdot \sqrt{4-x}}$

e. $k(x) = 5 \cdot \sqrt{2x^4+x^2} + 4x^2 = 5 \cdot \sqrt{u} + 4x^2 = 5 \cdot u^{\frac{1}{2}} + 4x^2$ met $u = 2x^4+x^2$ dus $u' = 8x^3+2x$

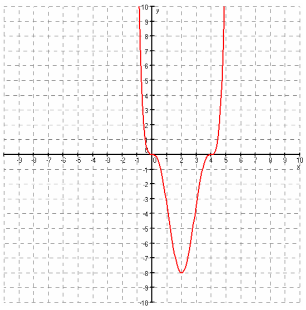
$k'(x) = 2\frac{1}{2}u^{-\frac{1}{2}} \cdot u' + 8x = \frac{5u'}{2\sqrt{u}} + 8x = \frac{5(8x^3+2x)}{2\sqrt{2x^4+x^2}} + 8x = \frac{5(4x^3+x)}{\sqrt{2x^4+x^2}} + 8x$

f. $l(x) = \frac{x^2+4}{\sqrt{x^2+4}} = \frac{u}{\sqrt{u}} = \sqrt{u} = u^{\frac{1}{2}}$ met $u = x^2+4$ dus $u' = 2x$

$l'(x) = \frac{1}{2}u^{-\frac{1}{2}} \cdot u' = \frac{1}{2u^{\frac{1}{2}}} \cdot u' = \frac{u'}{2 \cdot \sqrt{u}} = \frac{2x}{2 \cdot \sqrt{x^2+4}} = \frac{x}{\sqrt{x^2+4}}$

Opgave 29:

a.



b. $f(x) = (\frac{1}{2}x^2 - 2x)^3 = u^3$ met $u = \frac{1}{2}x^2 - 2x$ dus $u' = x-2$

$f'(x) = 3u^2 \cdot u' = 3(\frac{1}{2}x^2 - 2x)^2 \cdot (x-2) = 0$

$\frac{1}{2}x^2 - 2x = 0 \quad \vee \quad x - 2 = 0$

$\frac{1}{2}x(x-4) = 0 \quad \vee \quad x = 2$

$x = 0 \quad \vee \quad x = 2 \quad \vee \quad x = 4$

c. $y_A = f(6) = 216$

$rc = f'(6) = 432$

$y = 432x + b$ door $(6,216)$

$216 = 2592 + b$

$-2376 = b$

$y = 432x - 2376$

Opgave 30:

a. $f(x)=\sqrt{x^2+9}-x^2+5x$

$f'(x)=\frac{1}{2\sqrt{x^2+9}}\cdot 2x-2x+5=\frac{x}{\sqrt{x^2+9}}-2x+5$

$rc=f'(4)=-2\frac{1}{5}$

$y_A=f(4)=9$

$y=-2\frac{1}{5}x+b$ door $(4,9)$

$9=-8\frac{4}{5}+b$

$17\frac{4}{5}=b$

$y=-2\frac{1}{5}x+17\frac{4}{5}$

b. $f'(3)=\frac{3}{\sqrt{18}}-6+5\neq 0$ dus niet

c. $f'(x)=\frac{x}{\sqrt{x^2+9}}-2x+5=5$

$\frac{x}{\sqrt{x^2+9}}=2x$

$2x\sqrt{x^2+9)}=x$

$x=0 \quad \vee \quad 2\sqrt{x^2+9}=1$

$x=0 \quad \vee \quad \sqrt{x^2+9}=\frac{1}{2}$

$x=0 \quad \vee \quad x^2+9=\frac{1}{4}$

$x=0 \quad \vee \quad x^2=-8\frac{3}{4}$ kan niet

$y=f(0)=3$

$y=5x+b$ door $(0,3)$

$3+0=b$

$b=3$

$y=5x+3$

Opgave 31:

$f(x)=x\cdot\sqrt{2x+1}=g(x)\cdot h(x)$ met $g(x)=x$ en $h(x)=\sqrt{2x+1}$

$h(x)=\sqrt{2x+1}=\sqrt{u}$ met $u=2x+1$

Opgave 32:

a. $f(x)=x\cdot\sqrt{3x+1}$`

$f'(x)=1\cdot\sqrt{3x+1}+x\cdot\frac{1}{2\cdot\sqrt{3x+1}}\cdot 3=\sqrt{3x+1}+\frac{3x}{2\cdot\sqrt{3x+1}}=\frac{2(3x+1)}{2\cdot\sqrt{3x+1}}+\frac{3x}{2\cdot\sqrt{3x+1}}$

$=\frac{6x+2+3x}{2\cdot\sqrt{3x+1}}=\frac{9x+2}{2\cdot\sqrt{3x+1}}$

b. $g(x)=\frac{\sqrt{x^2+1}}{2x+1}$

$$g'(x)=\frac{(2x+1)\cdot\dfrac{1}{2\cdot\sqrt{x^2+1}}\cdot 2x-\sqrt{x^2+1}\cdot 2}{(2x+1)^2}=\frac{\dfrac{x(2x+1)}{\sqrt{x^2+1}}-2\cdot\sqrt{x^2+1}}{(2x+1)^2}=$$

$$=\frac{\dfrac{x(2x+1)}{\sqrt{x^2+1}}-\dfrac{2(x^2+1)}{\sqrt{x^2+1}}}{(2x+1)^2}=\frac{x(2x+1)-2(x^2+1)}{(2x+1)^2\cdot\sqrt{x^2+1}}=\frac{2x^2+x-2x^2-2}{(2x+1)^2\cdot\sqrt{x^2+1}}=$$

$$=\frac{x-2}{(2x+1)^2\cdot\sqrt{x^2+1}}$$

c. $h(x)=x\cdot(3x+1)^3$

$h'(x)=1\cdot(3x+1)^3+9x(3x+1)^2=(3x+1)^3+9x(3x+1)^2$

d. $k(x)=\dfrac{x^2-1}{\sqrt{4x+1}}$

$$k'(x)=\frac{\sqrt{4x+1}\cdot 2x-(x^2-1)\cdot\dfrac{1}{2\cdot\sqrt{4x+1}}\cdot 4}{4x+1}=\frac{2x\cdot\sqrt{4x+1}-\dfrac{2(x^2-1)}{\sqrt{4x+1}}}{4x+1}=$$

$$=\frac{\dfrac{2x(4x+1)}{\sqrt{4x+1}}-\dfrac{2(x^2-1)}{\sqrt{4x+1}}}{4x+1}=\frac{2x(4x+1)-2(x^2-1)}{(4x+1)\cdot\sqrt{4x+1}}=\frac{8x^2+2x-2x^2+2}{(4x+1)\cdot\sqrt{4x+1}}=$$

$$=\frac{6x^2+2x+2}{(4x+1)\cdot\sqrt{4x+1}}$$

Opgave 33:

$f(x)=\frac{1}{2}x\cdot\sqrt{3x+1}$

$y_A=f(8)=20$

$f'(x)=\frac{1}{2}\cdot\sqrt{3x+1}+\frac{1}{2}x\cdot\dfrac{1}{2\cdot\sqrt{3x+1}}\cdot 3=\frac{1}{2}\cdot\sqrt{3x+1}+\dfrac{3x}{4\cdot\sqrt{3x+1}}$

$rc=f'(8)=3\frac{7}{10}$

$y=3\frac{7}{10}x+b$ door $(8,20)$

$20=29\frac{3}{5}+b$

$-9\frac{3}{5}=b$

$y=3\frac{7}{10}x-9\frac{3}{5}$

Opgave 34:

$f(x)=\dfrac{x+1}{\sqrt{x^2+4}}$

a. $$f'(x)=\frac{\sqrt{x^2+4}\cdot 1-(x+1)\cdot\dfrac{1}{2\sqrt{x^2+4}}\cdot 2x}{x^2+4}=\frac{\sqrt{x^2+4}-\dfrac{x(x+1)}{\sqrt{x^2+4}}}{x^2+4}=$$

$$= \frac{\frac{x^2+4}{\sqrt{x^2+4}} - \frac{x^2+x}{\sqrt{x^2+4}}}{x^2+4} = \frac{\frac{x^2+4-x^2-x}{\sqrt{x^2+4}}}{x^2+4} = \frac{4-x}{(x^2+4)\cdot\sqrt{x^2+4}} = 0$$

$4 - x = 0$

$x = 4$

$y = f(4) = \frac{5}{\sqrt{20}} = \frac{5}{2\cdot\sqrt{5}} = \frac{1}{2}\sqrt{5}$ dus $(4, \frac{1}{2}\sqrt{5})$

b. $A = (0, \frac{1}{2})$

$rc = f'(0) = \frac{1}{2}$

$y = \frac{1}{2}x + b$ door $(0, \frac{1}{2})$

$\frac{1}{2} = 0 + b$

$b = \frac{1}{2}$

$y = \frac{1}{2}x + \frac{1}{2}$

snijpunt x-as: $\frac{1}{2}x + \frac{1}{2} = 0$

$\frac{1}{2}x = -\frac{1}{2}$

$x = -1$

$B = (-1, 0)$

$Opp(\Delta OAB) = \frac{1}{2}\cdot OB \cdot OA = \frac{1}{2}\cdot 1 \cdot \frac{1}{2} = \frac{1}{4}$

Opgave 35:

a. $g(x) = \frac{x+6}{\sqrt{8x+9}} = (x+6)\cdot(8x+9)^{-\frac{1}{2}} = h(x)\cdot j(x)$ met $h(x) = x+6$ en $j(x) = (8x+9)^{-\frac{1}{2}}$

dus $j(x) = (8x+9)^{-\frac{1}{2}} = u^{-\frac{1}{2}}$ met $u = 8x+9$ dus $u' = 8$

$j'(x) = -\frac{1}{2}u^{-1\frac{1}{2}}\cdot u' = -\frac{1}{2}\cdot(8x+9)^{-1\frac{1}{2}}\cdot 8$

$g'(x) = h'(x)\cdot j(x) + h(x)\cdot j'(x) = 1\cdot(8x+9)^{-\frac{1}{2}} + (x+6)\cdot -\frac{1}{2}(8x+9)^{-1\frac{1}{2}}\cdot 8 =$

$$= \frac{1}{\sqrt{8x+9}} - \frac{4(x+6)}{(8x+9)\sqrt{8x+9}} = \frac{8x+9}{(8x+9)\sqrt{8x+9}} - \frac{4x+24}{(8x+9)\sqrt{8x+9}} =$$

$$= \frac{8x+9-4x-24}{(8x+9)\sqrt{8x+9}} = \frac{4x-15}{(8x+9)\sqrt{8x+9}}$$

b. $f(x) = \frac{x+1}{\sqrt{x^2+4}} = (x+1)\cdot(x^2+4)^{-\frac{1}{2}}$

$$f'(x) = 1\cdot(x^2+4)^{-\frac{1}{2}} + (x+1)\cdot -\frac{1}{2}(x^2+4)^{-1\frac{1}{2}}\cdot 2x = \frac{1}{\sqrt{x^2+4}} - \frac{x(x+1)}{(x^2+4)\sqrt{x^2+4}} =$$

$$\frac{x^2+4}{(x^2+4)\sqrt{x^2+4}} - \frac{x^2+x}{(x^2+4)\sqrt{x^2+4}} = \frac{x^2+4-x^2-x}{(x^2+4)\sqrt{x^2+4}} = \frac{4-x}{(x^2+4)\sqrt{x^2+4}}$$

7.4 Raaklijnen en toppen

Opgave 36:

a. $5-2\sqrt{x}=2$

$-2\sqrt{x}=-3$

$\sqrt{x}=1\tfrac{1}{2}$

$x=2\tfrac{1}{4}$

b. $6+x\sqrt{x}=10$

$x\sqrt{x}=4$

$x^3=16$

$x=\sqrt[3]{16}$

c. $\dfrac{5x^2-10}{x^2-4}=0$

$5x^2-10=0$

$5x^2=10$

$x^2=2$

$x=\sqrt{2} \quad \vee \quad x=-\sqrt{2}$

d. $x^4-5x^2+4=0$

stel $p=x^2$ dan $p^2-5p+4=0$

$(p-1)(p-4)=0$

$p=1 \quad \vee \quad p=4$

$x^2=1 \quad \vee \quad x^2=4$

$x=1 \quad \vee \quad x=-1 \quad \vee \quad x=2 \quad \vee \quad x=-2$

e. $x^3-8x\sqrt{x}+12=0$

stel $p=x\sqrt{x}$ dan $p^2-8p+12=0$

$(p-2)(p-6)=0$

$p=2 \quad \vee \quad p=6$

$x\sqrt{x}=2 \quad \vee \quad x\sqrt{x}=6$

$x^3=4 \quad \vee \quad x^3=36$

$x=\sqrt[3]{4} \quad \vee \quad x=\sqrt[3]{36}$

f. $\dfrac{5x^2-10}{(x^2-4)^2}=1\tfrac{2}{5}$

$\dfrac{5x^2-10}{(x^2-4)^2}=\tfrac{7}{5}$

$7(x^2-4)^2=5(5x^2-10)$

$7(x^4-8x^2+16)=25x^2-50$

$7x^4-56x^2+112=25x^2-50$

$7x^4-81x^2+162=0$

stel $p=x^2$ dan $7p^2-81p+162=0$

$$p = \frac{81 \pm \sqrt{2025}}{14} = \frac{81 \pm 45}{14}$$

$p = \frac{81+45}{14} = 9 \quad \vee \quad p = \frac{81-45}{14} = \frac{18}{7}$

$x^2 = 9 \quad \vee \quad x^2 = \frac{18}{7}$

$x = 3 \quad \vee \quad x = -3 \quad \vee \quad x = \sqrt{\frac{18}{7}} \quad \vee \quad x = -\sqrt{\frac{18}{7}}$

Opgave 37:

$f(x) = 6x - 2x\sqrt{x}$

$f'(x) = 6 - 3\sqrt{x} = 0$

$-3\sqrt{x} = -6$

$\sqrt{x} = 2$

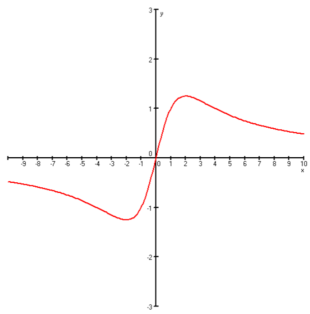
$x = 4$

$T(4,8)$

Opgave 38:

a. $f(x) = \dfrac{5x}{x^2+4}$

$$f'(x) = \frac{(x^2+4)\cdot 5 - 5x \cdot 2x}{(x^2+4)^2} = \frac{5x^2 + 20 - 10x^2}{(x^2+4)^2} = \frac{20 - 5x^2}{(x^2+4)^2} = 0$$



$20 - 5x^2 = 0$

$-5x^2 = -20$

$x^2 = 4$

$x = 2 \quad \vee \quad x = -2$

min $f(-2) = -1\frac{1}{4}$

max $f(2) = 1\frac{1}{4}$

$B_f = [-1\frac{1}{4}, 1\frac{1}{4}]$

b. $f'(x) = \dfrac{20 - 5x^2}{(x^2+4)^2} = \frac{3}{5}$

$3(x^2+4)^2 = 5(20 - 5x^2)$

$3(x^4 + 8x^2 + 16) = 100 - 25x^2$

$3x^4 + 24x^2 + 48 = 100 - 25x^2$

$3x^4 + 49x^2 - 52 = 0$

stel $p = x^2$ dan $3p^2 + 49p - 52 = 0$

$$p = \frac{-49 \pm \sqrt{3025}}{6} = \frac{-49 \pm 55}{6}$$

$p = \frac{-49+55}{6} = 1 \quad \vee \quad p = \frac{-49-55}{6} = -17\frac{1}{3}$

$x^2 = 1 \quad \vee \quad x^2 = -17\frac{1}{3}$

$x = 1 \quad \vee \quad x = -1$

Opgave 39:

a. $8 - 2x \geq 0$

$-2x \geq -8$
$x \leq 4$ dus $D_f = \langle \leftarrow, 4]$

b. $f'(x) = 1 \cdot \sqrt{8-2x} + x \cdot \dfrac{1}{2 \cdot \sqrt{8-2x}} \cdot -2 = \sqrt{8-2x} - \dfrac{x}{\sqrt{8-2x}}$

$= \dfrac{8-2x}{\sqrt{8-2x}} - \dfrac{x}{\sqrt{8-2x}} = \dfrac{8-3x}{\sqrt{8-2x}}$

c. $f'(x) = \dfrac{8-3x}{\sqrt{8-2x}} = 0$

$8-3x = 0$
$-3x = -8$
$x = 2\frac{2}{3}$
$y = 2\frac{2}{3} \cdot \sqrt{2\frac{2}{3}} = \frac{8}{3} \cdot \sqrt{\frac{24}{9}} = \frac{8}{3} \cdot \frac{2}{3} \cdot \sqrt{6} = \frac{16}{9}\sqrt{6}$
top= $(2\frac{2}{3}, 1\frac{7}{9}\sqrt{6})$
$B_f = \langle \leftarrow, 1\frac{7}{9}\sqrt{6}]$

d. $\dfrac{8-3x}{\sqrt{8-2x}} = 1$

$8-3x = \sqrt{8-2x}$
$9x^2 - 48x + 64 = 8 - 2x$
$9x^2 - 46x + 56 = 0$
$x = \dfrac{-46 \pm \sqrt{100}}{18} = \dfrac{46 \pm 10}{18}$
$x = \frac{46-10}{18} = 2 \quad \vee \quad x = \frac{46+10}{18} = 3\frac{1}{9}$ (vervalt)
$A(2,4)$

Opgave 40:

a. $y_A = f(0) = -3$

$f'(x) = 2 \cdot \sqrt{9-2x} + 2x \cdot \dfrac{1}{2 \cdot \sqrt{9-2x}} \cdot -2 = 2 \cdot \sqrt{9-2x} - \dfrac{2x}{\sqrt{9-2x}}$

$rc = f'(0) = 6$
$y = 6x + b$ door $(0,-3)$
$b = -3$
$y = 6x - 3$

b. $f'(x) = 2 \cdot \sqrt{9-2x} - \dfrac{2x}{\sqrt{9-2x}} = 0$

$2 \cdot \sqrt{9-2x} = \dfrac{2x}{\sqrt{9-2x}}$

$2(9-2x) = 2x$
$18 - 4x = 2x$
$-6x = -18$
$x = 3$
max $f(3) = 6\sqrt{3} - 3$

c. $9-2x \geq 0$

$-2x \geq -9$

$x \leq 4\frac{1}{2}$

$D_f = \langle \leftarrow, 4\frac{1}{2}]$

$B_f = \langle \leftarrow, 6\sqrt{3}-3]$

d. $f'(x) = 2\sqrt{9-2x} - \dfrac{2x}{\sqrt{9-2x}} = 1\frac{1}{2}$

$2(9-2x) - 2x = 1\frac{1}{2}\sqrt{9-2x}$

$18-4x-2x = 1\frac{1}{2}\sqrt{9-2x}$

$18-6x = 1\frac{1}{2}\sqrt{9-2x}$

$12-4x = \sqrt{9-2x}$

$16x^2 - 96x + 144 = 9-2x$

$16x^2 - 94x + 135 = 0$

$x = \dfrac{94 \pm \sqrt{196}}{32} = \dfrac{94 \pm 14}{32}$

$x = \frac{94-14}{32} = 2\frac{1}{2} \quad \vee \quad x = \frac{94+14}{32} = 3\frac{3}{8}$ (vervalt)

$B(2\frac{1}{2}, 7)$

Opgave 41:

a. $f(x) = \dfrac{x^3+2}{\sqrt{x}} = x^{2\frac{1}{2}} + 2x^{-\frac{1}{2}}$

$f'(x) = 2\frac{1}{2}x^{1\frac{1}{2}} - x^{-1\frac{1}{2}} = 2\frac{1}{2}x\sqrt{x} - \dfrac{1}{x\sqrt{x}} = \dfrac{2\frac{1}{2}x^3}{x\sqrt{x}} - \dfrac{1}{x\sqrt{x}} = \dfrac{2\frac{1}{2}x^3 - 1}{x\sqrt{x}} = 0$

$2\frac{1}{2}x^3 - 1 = 0$

$2\frac{1}{2}x^3 = 1$

$x^3 = \frac{2}{5}$

$x = \sqrt[3]{\frac{2}{5}}$

$y = \dfrac{2\frac{2}{5}}{\sqrt{\sqrt[3]{\frac{2}{5}}}} = \dfrac{2\frac{2}{5}}{\sqrt{(\frac{2}{5})^{\frac{1}{3}}}} = \dfrac{2\frac{2}{5}}{((\frac{2}{5})^{\frac{1}{3}})^{\frac{1}{2}}} = \dfrac{2\frac{2}{5}}{(\frac{2}{5})^{\frac{1}{6}}} = \dfrac{2\frac{2}{5}}{\sqrt[6]{\frac{2}{5}}}$

$a = 2\frac{2}{5} \quad b = 6 \quad c = \frac{2}{5}$

b. $\dfrac{2\frac{1}{2}x^3 - 1}{x\sqrt{x}} = 1\frac{1}{2}$

$2\frac{1}{2}x^3 - 1 = 1\frac{1}{2}x\sqrt{x}$

$5x^3 - 2 = 3x\sqrt{x}$

$5x^3 - 3x\sqrt{x} - 2 = 0$

stel $p = x\sqrt{x}$ dan $5p^2 - 3p - 2 = 0$

$p = \dfrac{3 \pm \sqrt{49}}{10} = \dfrac{3 \pm 7}{10}$

$p = \frac{3+7}{10} = 1 \quad \vee \quad p = \frac{3-7}{10} = -\frac{2}{5}$

$x\sqrt{x}=1 \quad \vee \quad x\sqrt{x}=-\frac{2}{5}$ (kan niet)

$x^3=1$

$x=1$

$y=3$

$y=1\frac{1}{2}x+b$ door $(1,3)$

$3=1\frac{1}{2}+b$

$b=1\frac{1}{2}$

$y=1\frac{1}{2}x+1\frac{1}{2}$

Opgave 42:

a. $f'(x)=\dfrac{(x\sqrt{x}+1)\cdot 9-9x\cdot 1\frac{1}{2}\sqrt{x}}{(x\sqrt{x}+1)^2}=\dfrac{9x\sqrt{x}+9-13\frac{1}{2}x\sqrt{x}}{(x\sqrt{x}+1)^2}=\dfrac{9-4\frac{1}{2}x\sqrt{x}}{(x\sqrt{x}+1)^2}$

$f'(4)=-\frac{1}{3}$

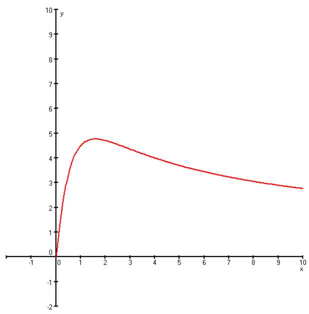
$y=-\frac{1}{3}x+b$ door $(4,4)$

$4=-\frac{4}{3}+b$

$b=5\frac{1}{3}$

$y=-\frac{1}{3}x+5\frac{1}{3}$

b. $f'(x)=\dfrac{9-4\frac{1}{2}x\sqrt{x}}{(x\sqrt{x}+1)^2}=0$



$9-4\frac{1}{2}x\sqrt{x}=0$

$-4\frac{1}{2}x\sqrt{x}=-9$

$x\sqrt{x}=2$

$x^3=4$

$x=\sqrt[3]{4}=\sqrt[3]{2^2}=2^{\frac{2}{3}}$

$\max\ f(\sqrt[3]{4})=\dfrac{9\cdot\sqrt[3]{4}}{2^{\frac{2}{3}}\cdot\sqrt{2^{\frac{2}{3}}}+1}=\dfrac{9\cdot\sqrt[3]{4}}{2^{\frac{2}{3}}\cdot 2^{\frac{1}{3}}+1}=\dfrac{9\cdot\sqrt[3]{4}}{2+1}=\dfrac{9\cdot\sqrt[3]{4}}{3}=3\cdot\sqrt[3]{4}$

c. $f'(x)=\dfrac{9-4\frac{1}{2}x\sqrt{x}}{(x\sqrt{x}+1)^2}=\frac{9}{8}$

$9(x\sqrt{x}+1)^2=8(9-4\frac{1}{2}x\sqrt{x})$

$(x\sqrt{x}+1)^2=8(1-\frac{1}{2}x\sqrt{x})$

$x^3+2x\sqrt{x}+1=8-4x\sqrt{x}$

$x^3+6x\sqrt{x}-7=0$

stel $p=x\sqrt{x}$ dan $p^2+6p-7=0$

$(p-1)(p+7)=0$

$p=1 \quad \vee \quad p=-7$

$x\sqrt{x}=1 \quad \vee \quad x\sqrt{x}=-7$ (kan niet)

$x^3=1$

$x=1$ dus $B(1,4\frac{1}{2})$

Opgave 43:

a. $\frac{dN}{dt} = 90 - 60\sqrt{t}$

$\left[\frac{dN}{dt}\right]_{t=1} = 30$

Om 8 uur neemt het aantal auto's dat per minuut passeert toe met 30 per uur.

b. $\frac{dN}{dt} = 90 - 60\sqrt{t} = 0$

$-60\sqrt{t} = -90$

$\sqrt{t} = 1{,}5$

$t = 2{,}25$ dus om 9.15 uur

c. de afname is 1 per 2 minuten, dus 30 per uur

$\frac{dN}{dt} = 90 - 60\sqrt{t} = -30$

$-60\sqrt{t} = -120$

$\sqrt{t} = 2$

$t = 4$ dus om 11.00 uur

Opgave 44:

a. $f_2(x) = x^3 + 2x^2$ dus $f_2'(x) = 3x^2 + 4x$

$f_5(x) = x^3 + 5x^2$ dus $f_5'(x) = 3x^2 + 10x$

b. $f_p'(x) = 3x^2 + 2px$

Opgave 45:

$f_p'(x) = -\frac{1}{2}x + p = 0$

$p = \frac{1}{2}x$

Opgave 46:

$f_p'(x) = x^2 + 2px = 0$

$2px = -x^2$

$p = -\frac{1}{2}x$ voor $x \neq 0$; $x = 0$ geeft $y = 5$

$y = \frac{1}{3}x^3 - \frac{1}{2}x \cdot x^2 + 5$

$y = -\frac{1}{6}x^3 + 5$ ook $(0,5)$ ligt op deze grafiek

Opgave 47:

$$f_p'(x) = \frac{(x^2+4)\cdot p - px \cdot 2x}{(x^2+4)^2} = \frac{px^2 + 4p - 2px^2}{(x^2+4)^2} = \frac{4p - px^2}{(x^2+4)^2} = 0$$

$4p - px^2 = 0$

$p(4 - x^2) = 0$

$p = 0 \quad \vee \quad x^2 = 4$

dus $x = 2 \quad \vee \quad x = -2$

als $p = 0$ geldt $f(x) = 0$; dit is een horizontale lijn , dus geen toppen

Opgave 48:

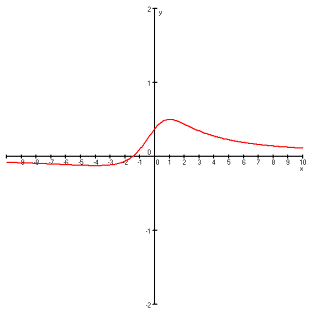
a. $f_p'(x)=\frac{(x^2+4)\cdot 1-(x+p)\cdot 2x}{(x^2+4)^2}=\frac{x^2+4-2x^2-2px}{(x^2+4)^2}=\frac{-x^2-2px+4}{(x^2+4)^2}$

$f_p'(1)=\frac{-2p+3}{25}=0$

$-2p+3=0$

$-2p=-3$

$p=1\frac{1}{2}$



$f_{1\frac{1}{2}}'(x)=\frac{-x^2-3x+4}{(x^2+4)^2}=0$

$-x^2-3x+4=0$

$x^2+3x-4=0$

$(x+4)(x-1)=0$

$x=-4 \quad \vee \quad x=1$

min $f(-4)=-\frac{1}{8}$

b. $f_p'(x)=\frac{-x^2-2px+4}{(x^2+4)^2}=0$

$-x^2-2px+4=0$

$-2px=x^2-4$

$p=-\frac{1}{2}x+\frac{2}{x}$ voor $x\neq 0$; $x=0$ geeft $y=\frac{1}{4}p$

$y=\frac{x-\frac{1}{2}x+\frac{2}{x}}{x^2+4}=\frac{\frac{1}{2}x+\frac{2}{x}}{x^2+4}=\frac{\frac{x^2}{2x}+\frac{4}{2x}}{x^2+4}=\frac{\frac{x^2+4}{2x}}{x^2+4}=\frac{1}{2x}$

7.5 Toppen en snijpunten

Opgave 49:

één oplossing
twee oplossingen

Opgave 50:

a. $f'(x) = 6x^2 - 6x - 36 = 0$
$x^2 - x - 6 = 0$
$(x+2)(x-3) = 0$
$x = -2 \quad \vee \quad x = 3$
$(-2{,}54)$ en $(3{,}-71)$

b. drie oplossingen
één oplossing

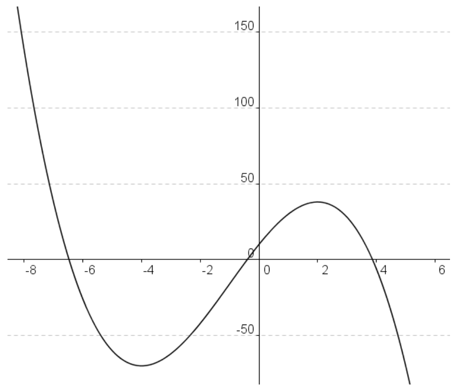
c. de horizontale lijnen door de toppen hebben twee snijpunten met de grafiek van *f*, daar tussen hebben de horizontale lijnen drie snijpunten met de grafiek van *f*.

d. $p < -71 \quad \vee \quad p > 54$

e. $p = -71 \quad \vee \quad p = 54$

Opgave 51:

a. $f'(x) = -3x^2 - 6x + 24 = 0$
$x^2 + 2x - 8 = 0$
$(x+4)(x-2) = 0$
$x = -4 \quad \vee \quad x = 2$
min $f(-4) = -70$
max $f(2) = 38$



b. drie oplossingen
één oplossing

c. $-70 < p < 38$

d. $p < -70 \quad \vee \quad p > 38$

Opgave 52:

$f'(x) = 3x^3 - 6x^2 - 72x = 0$
$3x(x^2 - 2x - 24) = 0$
$x(x+4)(x-6) = 0$
$x = 0 \quad \vee \quad x = -4 \quad \vee \quad x = 6$
$(-4{,}44)$ en $(0{,}300)$ en $(6{,}-456)$
vier oplossingen: $44 < p < 300$
drie oplossingen: $p = 44 \quad \vee \quad p = 300$
twee oplossingen: $-456 < p < 44 \quad \vee \quad p > 300$
één oplossing: $p = -456$
geen oplossingen: $p < -456$

Opgave 53:

a. $f'(x) = 2x \cdot \sqrt{2x+5} + x^2 \cdot \frac{1}{2\sqrt{2x+5}} \cdot 2 = 2x \cdot \sqrt{2x+5} + \frac{x^2}{\sqrt{2x+5}} =$

$$= \frac{2x(2x+5)}{\sqrt{2x+5}} + \frac{x^2}{\sqrt{2x+5}} = \frac{4x^2+10x}{\sqrt{2x+5}} + \frac{x^2}{\sqrt{2x+5}} = \frac{5x^2+10x}{\sqrt{2x+5}} = 0$$

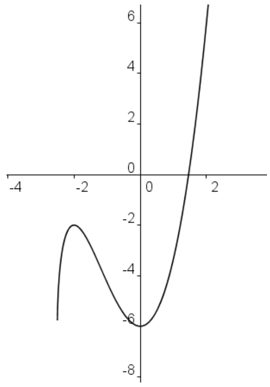
$5x^2 + 10x = 0$

$5x(x+2) = 0$

$x = 0 \quad \vee \quad x = -2$

max $f(-2) = -2$

min $f(0) = -6$



b. de grafiek begint in het punt $(-2\tfrac{1}{2}, -6)$

geen oplossing: $p < -6$

één oplossingen: $p > -2$

twee oplossingen: $p = -6 \quad \vee \quad p = -2$

drie oplossingen: $-6 < p < -2$

Opgave 54:

a. de lijn $y = \tfrac{3}{4}x + 1$ ligt boven de hoogste raaklijn, dus deze lijn heeft één snijpunt met de grafiek van *f*.

b. $-\tfrac{3}{4} < p < \tfrac{3}{4}$

Opgave 55:

a. $f'(x) = \frac{(x-3)\cdot 1 - (x-2)\cdot 1}{(x-3)^2} = \frac{x-3-x+2}{(x-3)^2} = \frac{-1}{(x-3)^2}$

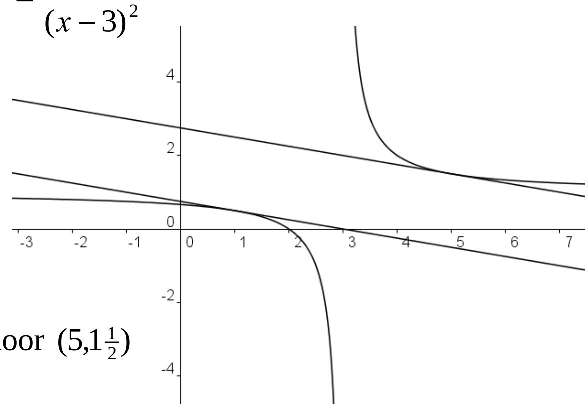
$\frac{-1}{(x-3)^2} = -\tfrac{1}{4}$

$(x-3)^2 = 4$

$x - 3 = -2 \quad \vee \quad x - 3 = 2$

$x = 1 \quad \vee \quad x = 5$

$y = \tfrac{1}{2} \qquad y = 1\tfrac{1}{2}$



$y = -\tfrac{1}{4}x + b$ door $(1, \tfrac{1}{2})$ | $y = -\tfrac{1}{4}x + b$ door $(5, 1\tfrac{1}{2})$

$\tfrac{1}{2} = -\tfrac{1}{4} + b$ | $1\tfrac{1}{2} = -1\tfrac{1}{4} + b$

$b = \tfrac{3}{4}$ | $b = 2\tfrac{3}{4}$

$y = -\tfrac{1}{4}x + \tfrac{3}{4}$ | $y = -\tfrac{1}{4}x + 2\tfrac{3}{4}$

b. $\tfrac{3}{4} < p < 2\tfrac{3}{4}$

Opgave 56:

a. $f'(x) = x - 1\tfrac{1}{2}\sqrt{x}$

$x - 1\tfrac{1}{2}\sqrt{x} = 4\tfrac{1}{2}$

$x - 4\tfrac{1}{2} = 1\tfrac{1}{2}\sqrt{x}$

$2x - 9 = 3\sqrt{x}$

$4x^2 - 36x + 81 = 9x$

$4x^2 - 45x + 81 = 0$

$x = \frac{45 \pm \sqrt{729}}{8} = \frac{45 \pm 27}{8}$

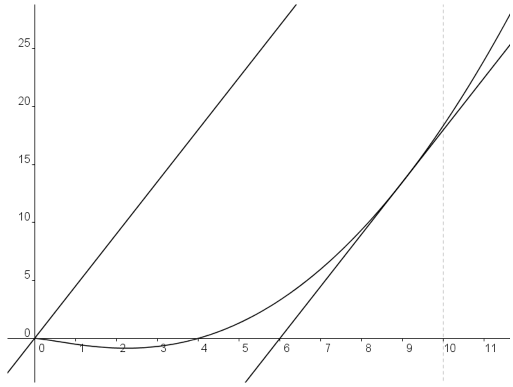
$x = \frac{45+27}{8} = 9 \quad \vee \quad x = \frac{45-27}{8} = 2\frac{1}{4}$ (vervalt)

$y = 4\frac{1}{2}x + b$ door $(9{,}13\frac{1}{2})$

$13\frac{1}{2} = 40\frac{1}{2} + b$

$b = -27$

$y = 4\frac{1}{2}x - 27$



b. $-27 < p \leq 0$

Opgave 57:

a. drie oplossingen
één oplossing

b. de lijn $y = ax$ gaat door $(0{,}0)$ en ligt tussen de x-as en lijn k. Deze lijn snijdt de grafiek van f behalve in $(0{,}0)$ ook nog in twee andere punten, dus de vergelijking heeft in totaal drie oplossingen.

Opgave 58:

a. $f'(x) = \frac{(x^2+5)\cdot 6 - 6x \cdot 2x}{(x^2+5)^2} = \frac{6x^2 + 30 - 12x^2}{(x^2+5)^2} = \frac{-6x^2+30}{(x^2+5)^2} = 0$

$-6x^2 + 30 = 0$

$-6x^2 = -30$

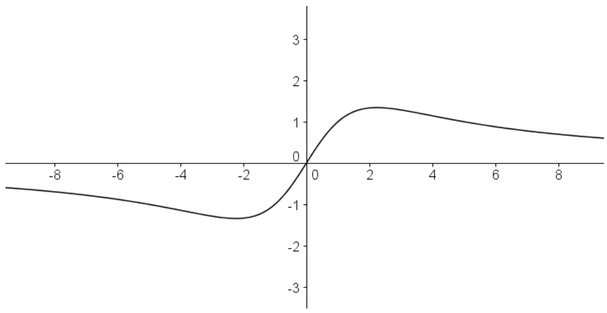
$x^2 = 5$

$x = -\sqrt{5} \quad \vee \quad x = \sqrt{5}$

min $f(-\sqrt{5}) = -\frac{3}{5}\sqrt{5}$

max $f(\sqrt{5}) = \frac{3}{5}\sqrt{5}$

$B_f = [-\frac{3}{5}\sqrt{5}, \frac{3}{5}\sqrt{5}]$



b. $f'(0) = 1\frac{1}{5}$

dus $a \geq 1\frac{1}{5} \quad \vee \quad a \leq 0$

c. $f'(x) = \frac{-6x^2+30}{(x^2+5)^2} = \frac{2}{3}$

$2(x^2+5)^2 = 3(-6x^2+30)$

$(x^2+5)^2 = 3(-3x^2+15)$

$x^4 + 10x^2 + 25 = -9x^2 + 45$

$x^4 + 19x^2 - 20 = 0$

$(x^2+20)(x^2-1) = 0$

$x^2 = -20 \quad \vee \quad x^2 = 1$

$x = 1 \quad \vee \quad x = -1$

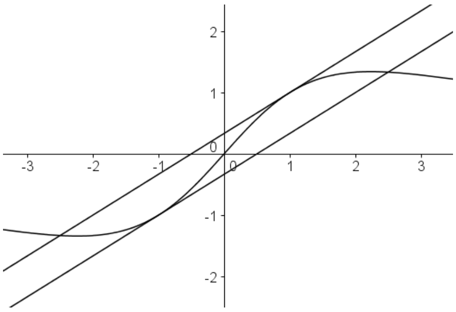
$y = 1 \qquad\quad y = -1$

$y = \frac{2}{3}x + p$ door $(1{,}1)$ en $y = \frac{2}{3}x + p$ door $(-1{,}-1)$

$1 = \frac{2}{3} + p$ $\qquad$ $-1 = -\frac{2}{3} + p$

$p = \frac{1}{3}$ $\qquad$ $p = -\frac{1}{3}$

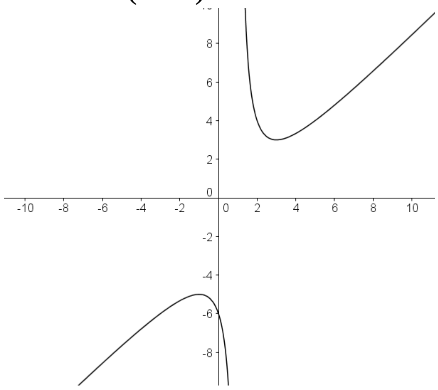
dus $-\frac{1}{3} < p < \frac{1}{3}$



Opgave 59:

a. $f'(x) = \dfrac{(x-1)(2x-3) - (x^2 - 3x + 6)\cdot 1}{(x-1)^2} = \dfrac{2x^2 - 5x + 3 - x^2 + 3x - 6}{(x-1)^2} =$

$= \dfrac{x^2 - 2x - 3}{(x-1)^2} = 0$



$x^2 - 2x - 3 = 0$

$(x+1)(x-3) = 0$

$x = -1 \quad \vee \quad x = 3$

max $f(-1) = -5$

min $f(3) = 3$

$p \leq -5 \quad \vee \quad p \geq 3$

b. $f'(x) = \dfrac{x^2 - 2x - 3}{(x-1)^2} = \frac{5}{9}$

$9(x^2 - 2x - 3) = 5(x-1)^2$

$9x^2 - 18x - 27 = 5x^2 - 10x + 5$

$4x^2 - 8x - 32 = 0$

$x^2 - 2x - 8 = 0$

$(x+2)(x-4) = 0$

$x = -2 \quad \vee \quad x = 4$

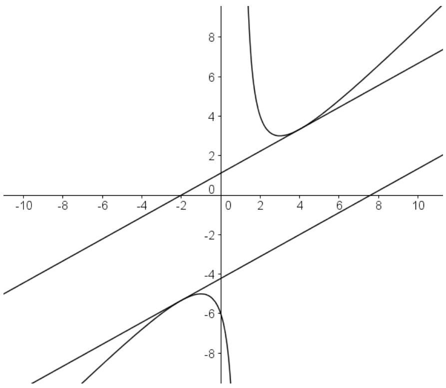
$y = -5\frac{1}{3}$ $\qquad$ $y = 3\frac{1}{3}$

$y = \frac{5}{9}x + q$ door $(-2,-5\frac{1}{3})$ $\qquad$ $y = \frac{5}{9}x + q$ door $(4,3\frac{1}{3})$

$-5\frac{1}{3} = -\frac{10}{9} + q$ $\qquad$ $3\frac{1}{3} = \frac{20}{9} + q$

$q = -\frac{38}{9}$ $\qquad$ $q = \frac{10}{9}$

dus $-\frac{38}{9} < q < \frac{10}{9}$



Opgave 60:

a. $f'(x) = 1\cdot\sqrt{2x+6} + x\cdot\dfrac{1}{2\sqrt{2x+6}}\cdot 2 = \sqrt{2x+6} + \dfrac{x}{\sqrt{2x+6}} = \dfrac{2x+6}{\sqrt{2x+6}} + \dfrac{x}{\sqrt{2x+6}} =$

$= \dfrac{3x+6}{\sqrt{2x+6}} = 0$

$3x + 6 = 0$

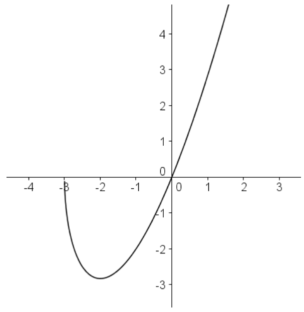
$3x = -6$

$x = -2$

top $(-2,-2\sqrt{2})$

beginpunt $(-3,0)$

dus $-2\sqrt{2} < p \le 0$



b. $\dfrac{3x+6}{\sqrt{2x+6}} = \frac{3}{2}$

$6x+12 = 3\sqrt{2x+6}$

$2x+4 = \sqrt{2x+6}$

$4x^2 + 16x + 16 = 2x+6$

$4x^2 + 14x + 10 = 0$

$x = \dfrac{-14 \pm \sqrt{36}}{8} = \dfrac{-14 \pm 6}{8}$

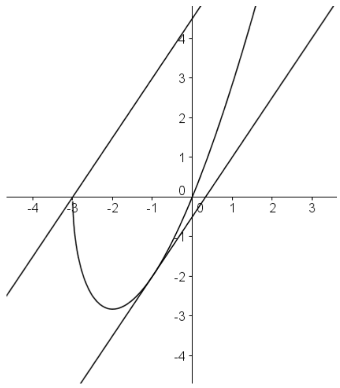
$x = \dfrac{-14-6}{8} = -2\frac{1}{2}$ (vervalt) $\quad \vee \quad x = \dfrac{-14+6}{8} = -1$

$y = -2$

dus $y = 1\frac{1}{2}x + q$ door $(-1,-2)$

$-2 = -1\frac{1}{2} + q$

$q = -\frac{1}{2}$



de lijn door $(-3,0)$ heeft ook twee snijpunten

$y = 1\frac{1}{2}x + q$ door $(-3,0)$

$0 = -4\frac{1}{2} + q$

$q = 4\frac{1}{2}$

dus $-\frac{1}{2} < x \le 4\frac{1}{2}$

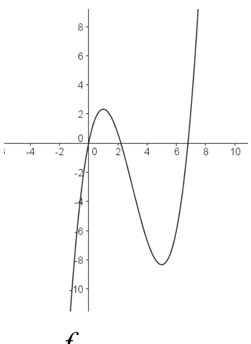
c. de lijn $y = ax$ gaat altijd door het punt $(0,0)$

$f'(0) = \sqrt{6}$

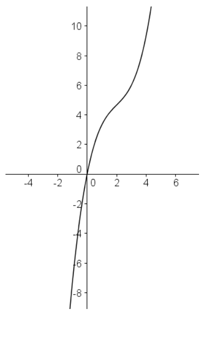
dus $0 \le a < \sqrt{6} \quad \vee \quad a > \sqrt{6}$

Opgave 61:

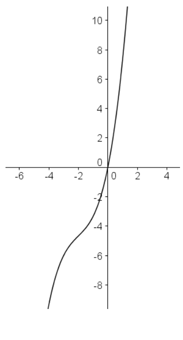
a.



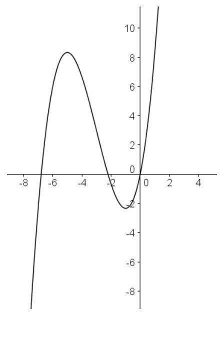
f_{-3}



f_{-2}



f_2



f_3

b. f_{-3} twee extreme waarden

f_{-2} geen extreme waarden

f_2 geen extreme waarden

f_3 twee extreme waarden

Opgave 62:

Voor $p > \frac{1}{2}$ geen, want dan geldt dat $D < 0$ dus geen extreme waarden.

Voor $p = \frac{1}{2}$ geen, want dan geldt $D = 0$, dan is er wel een punt waar de raaklijn horizontaal loopt, maar dit is geen extreme waarden, want de grafiek van *f* is verder overal dalend.

Opgave 63:

$f_p'(x) = -x^2 - 3x + p$

f_p heeft twee extreme waarden dus $f_p'(x) = 0$ heeft twee oplossingen

$D = 9 + 4p > 0$

$4p > -9$

$p > -2\frac{1}{4}$

Opgave 64:

$f_p'(x) = \frac{3}{4}x^2 + 2px + 3$

f_p heeft geen extreme waarden dus $f_p'(x) = 0$ heeft 1 of geen oplossing

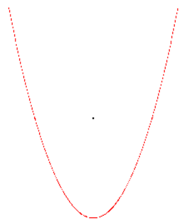
$D = 4p^2 - 9 \leq 0$

$4p^2 = 9$

$p^2 = 2\frac{1}{4}$

$p = -1\frac{1}{2} \quad \vee \quad p = 1\frac{1}{2}$

$-1\frac{1}{2} \leq p \leq 1\frac{1}{2}$



Opgave 65:

a. $f_p'(x) = \frac{1}{4}x^2 + 2x + p$

$f_p'(1) = \frac{1}{4} + 2 + p = 0$

$p = -2\frac{1}{4}$

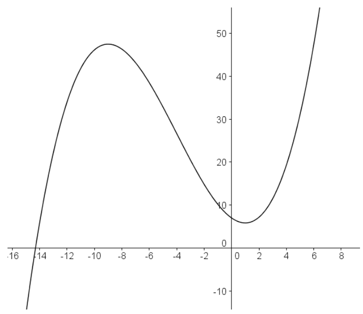
$f'(x) = \frac{1}{4}x^2 + 2x - 2\frac{1}{4} = 0$

$x^2 + 8x - 9 = 0$

$(x+9)(x-1) = 0$

$x = -9 \quad \vee \quad x = 1$

max $f(-9) = 47\frac{1}{2}$



b. f_p heeft twee extreme waarden, dus $f_p'(x) = 0$ heeft twee oplossingen, dus $D > 0$

$D = 4 - p > 0$

$-p > -4$

$p < 4$

Opgave 66:

a. $f'(x)$ geeft de richtingscoëfficiënt van de raaklijn, dus $f_p'(3) = 2$

b. $f_p'(x) = 9 + 6p + 5 = 2$

$6p = -12$

$p = -2$

Opgave 67:

a. $f_p'(x) = 9\sqrt{x} + 2px$

$f_p'(2\tfrac{1}{4}) = 13\tfrac{1}{2} + 4\tfrac{1}{2}p = 0$

$4\tfrac{1}{2}p = -13\tfrac{1}{2}$

$p = -3$

b. $f_p'(1) = 9 + 2p = 5$

$2p = -4$

$p = -2$

$f_{-2}(x) = 6x\sqrt{x} - 2x^2$

$f_{-2}'(1) = 4$

$y = 5x + q$ door $(1,4)$

$4 = 5 + q$

$q = -1$

Opgave 68:

a. snijpunt met de y-as: $(0,4)$

$f_p(0) = \dfrac{p}{1} = p = 4$

$p = 4$

$f_4(x) = \dfrac{4x+4}{x^2+1}$

$f_4'(x) = \dfrac{(x^2+1)\cdot 4 - (4x+4)\cdot 2x}{(x^2+1)^2} = \dfrac{4x^2+4-8x^2-8x}{(x^2+1)^2} = \dfrac{-4x^2-8x+4}{(x^2+1)^2}$

$f_4'(0) = 4 = a$ dus $a = 4$

b. $f_p'(x) = \dfrac{(x^2+1)\cdot 4 - (4x+p)\cdot 2x}{(x^2+1)^2} = \dfrac{4x^2+4-8x^2-2px}{(x^2+1)^2} = \dfrac{-4x^2+4-2px}{(x^2+1)^2}$

$f_p'(-1) = \dfrac{2p}{4} = \tfrac{1}{2}p = -1$

$p = -2$

$f_{-2}(x) = \dfrac{4x-2}{x^2+1}$

$f_{-2}(-1) = -3$

$y = -x + b$ door $(-1,-3)$

$-3 = 1 + b$

$b = -4$

$y = -x - 4$

c. $f_p'(x) = \dfrac{-12-4p}{25} = 0$

$-12 - 4p = 0$

$-4p = 12$

$p = -3$

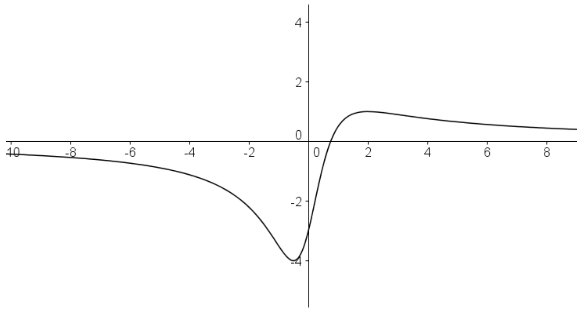
$f'_{-3}(x) = \dfrac{-4x^2 + 6x + 4}{(x^2+1)^2} = 0$

$-4x^2 + 6x + 4 = 0$

$x = \dfrac{-6 \pm \sqrt{100}}{-8} = \dfrac{-6 \pm 10}{-8}$

$x = \frac{-6-10}{-8} = 2 \quad \vee \quad x = \frac{-6+10}{-8} = -\frac{1}{2}$

min $f_{-3}(-\frac{1}{2}) = -4$



Opgave 69:

a. $f'_p(x) = x^2 + 2px - 3 = 0$

$D = 4p^2 + 12 > 0$ voor iedere waarde van p, dus $f'_p(x) = 0$ heeft altijd twee oplossingen, dus f_p heeft twee extreme waarden.

b. $f'_p(3) = 9 + 6p - 3 = 0$

$6p = -6$

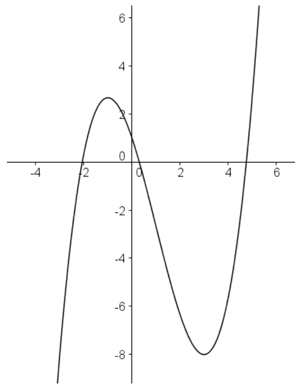
$p = -1$

$f'_{-1}(x) = x^2 - 2x - 3 = 0$

$(x+1)(x-3) = 0$

$x = -1 \quad \vee \quad x = 3$

max $f_{-1}(-1) = 2\frac{2}{3}$



c. $f'_p(-2) = 4 - 4p - 3 = -1$

$-4p = -2$

$p = \frac{1}{2}$

$f_{\frac{1}{2}}(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 3x - \frac{1}{2}$

$f_{\frac{1}{2}}(-2) = 4\frac{5}{6}$

$y = -x + q$ door $(-2, 4\frac{5}{6})$

$4\frac{5}{6} = 2 + q$

$q = 2\frac{5}{6}$

Opgave 70:

$$f'_p(x) = \frac{(x^2+2)\cdot 9 \cdot \dfrac{1}{2\sqrt{x^2+p}} \cdot 2x - 9\sqrt{x^2+p}\cdot 2x}{(x^2+2)^2} = \frac{\dfrac{9x(x^2+2)}{\sqrt{x^2+p}} - 18x\sqrt{x^2+p}}{(x^2+2)^2} =$$

$$= \frac{\dfrac{9x(x^2+2)}{\sqrt{x^2+p}} - \dfrac{18x(x^2+p)}{\sqrt{x^2+p}}}{(x^2+2)^2} = \frac{\dfrac{9x^3+18x-18x^3-18px}{\sqrt{x^2+p}}}{(x^2+2)^2} = \frac{-9x^3+18x-18px}{\sqrt{x^2+p}\cdot(x^2+2)^2}$$

$f'_p(-1) = \dfrac{-9+18p}{\sqrt{1+p}\cdot 9} = \dfrac{-1+2p}{\sqrt{1+p}} = 2\frac{1}{2}$

$2\frac{1}{2}\sqrt{1+p} = -1 + 2p$

$5\sqrt{1+p} = -2 + 4p$

$25(1+p) = 4-16p+16p^2$

$25+25p = 4-16p+16p^2$

$-16p^2+41p+21=0$

$p = \frac{-41 \pm \sqrt{3025}}{-32} = \frac{-41 \pm 55}{-32}$

$p = \frac{-41-55}{-32} = 3 \quad \vee \quad p = \frac{-41+55}{-32} = -\frac{7}{16}$ (vervalt)

$f_3(x) = \frac{9\sqrt{x^2+3}}{x^2+2}$

$f_3(-1) = 6$

$y = 2\frac{1}{2}x + b$ door $(-1,6)$

$6 = -2\frac{1}{2} + b$

$b = 8\frac{1}{2}$

$y = 2\frac{1}{2}x + 8\frac{1}{2}$

7.6 Diagnostische toets

Opgave 1:

a. $f'(x) = (2x+3)(3-7x) + (x^2+3x)\cdot -7 = (2x+3)(3-7x) - 7(x^2+3x)$

b. $g'(x) = 6x(3x^2+4) + (3x^2+4)\cdot 6x = 12x(3x^2+4)$

Opgave 2:

a. $f'(x) = \dfrac{(x^2+2)\cdot 3 - (3x-7)\cdot 2x}{(x^2+2)^2} = \dfrac{3x^2+6-6x^2+14x}{(x^2+2)^2} = \dfrac{-3x^2+14x+6}{(x^2+2)^2}$

b. $g'(x) = 3 - \dfrac{(x+4)\cdot 0 - 2\cdot 1}{(x+4)^2} = 3 + \dfrac{2}{(x+4)^2}$

Opgave 3:

$f(x) = \dfrac{x^2-9}{3x+2} = 0$

$x^2 - 9 = 0$

$x^2 = 9$

$x = -3 \quad \vee \quad x = 3$

$f'(x) = \dfrac{(3x+2)\cdot 2x - (x^2-9)\cdot 3}{(3x+2)^2} = \dfrac{6x^2+4x-3x^2+27}{(3x+2)^2} = \dfrac{3x^2+4x+27}{(3x+2)^2}$

$f'(-3) = \frac{6}{7}$	$f'(3) = \frac{6}{11}$
$y = \frac{6}{7}x + b$ door $(-3,0)$	$y = \frac{6}{11}x + b$ door $(3,0)$
$0 = -\frac{18}{7} + b$	$0 = \frac{18}{11} + b$
$b = \frac{18}{7} = 2\frac{4}{7}$	$b = -\frac{18}{11} = -1\frac{7}{11}$
$y = \frac{6}{7}x + 2\frac{4}{7}$	$y = \frac{6}{11}x - 1\frac{7}{11}$

Opgave 4:

a. $f(x) = \dfrac{2}{x^5} = 2x^{-5}$

$f'(x) = -10x^{-6} = -\dfrac{10}{x^6}$

b. $g(x) = \dfrac{x^5+2}{x^3} = \dfrac{x^5}{x^3} + \dfrac{2}{x^3} = x^2 + 2x^{-3}$

$g'(x) = 2x - 6x^{-4} = 2x - \dfrac{6}{x^4}$

c. $h(x) = \dfrac{3}{x} - \dfrac{x}{3} = 3x^{-1} - \frac{1}{3}x$

$h'(x) = -3x^{-2} - \frac{1}{3} = -\dfrac{3}{x^2} - \frac{1}{3}$

Opgave 5:

a. $f(x) = x^3 + \sqrt[3]{x^2} = x^3 + x^{\frac{2}{3}}$

$f'(x) = 3x^2 + \frac{2}{3}x^{-\frac{1}{3}} = 3x^2 + \dfrac{2}{3x^{\frac{1}{3}}} = 3x^2 + \dfrac{2}{3\cdot\sqrt[3]{x}}$

b. $g(x) = x^3 \cdot \sqrt[3]{x^2} = x^3 \cdot x^{\frac{2}{3}} = x^{3\frac{2}{3}}$

$g'(x) = 3\frac{2}{3}x^{2\frac{2}{3}} = 3\frac{2}{3}x^2 \cdot \sqrt[3]{x^2}$

c. $h(x) = \dfrac{x\sqrt{x}}{x^3+1}$

$h'(x) = \dfrac{(x^3+1)\cdot 1\frac{1}{2}\sqrt{x} - x\sqrt{x}\cdot 3x^2}{(x^3+1)^2} = \dfrac{1\frac{1}{2}x^3\cdot\sqrt{x} + 1\frac{1}{2}\sqrt{x} - 3x^2\cdot\sqrt{x}}{(x^3+1)^2} =$

$= \dfrac{-1\frac{1}{2}x^3\cdot\sqrt{x} + 1\frac{1}{2}\sqrt{x}}{(x^3+1)^2} = \dfrac{-3x^3\cdot\sqrt{x} + 3\sqrt{x}}{2(x^3+1)^2}$

Opgave 6:

$y_A = -2$

$f(x) = \dfrac{x^2-3}{x^2\sqrt{x}} = \dfrac{x^2-3}{x^{2\frac{1}{2}}} = x^{-\frac{1}{2}} - 3x^{-2\frac{1}{2}}$

$f'(x) = -\frac{1}{2}x^{-1\frac{1}{2}} + 7\frac{1}{2}x^{-3\frac{1}{2}} = -\dfrac{1}{2x\sqrt{x}} + \dfrac{15}{2x^3\cdot\sqrt{x}}$

$f'(1) = 7$

$y = 7x + b$ door $(1,-2)$

$-2 = 7 + b$

$b = -9$

$y = 7x - 9$

snijpunt x-as: $7x - 9 = 0$

$7x = 9$

$x = \frac{9}{7}$ dus $(\frac{9}{7},0)$

snijpunt y-as: $(0,-9)$

$Opp(\Delta)BC) = \frac{1}{2}\cdot\frac{9}{7}\cdot 9 = \frac{81}{14}$

Opgave 7:

a. $f(x) = 3(x^2+4x)^4 = 3u^4$ met $u = x^2 + 4x$ dus $u' = 2x + 4$

$f'(x) = 12u^3 \cdot u' = 12(x^2+4x)^3\cdot(2x+4)$

b. $g(x) = (x^2+2)\sqrt{x^2+2} = (x^2+2)^{1\frac{1}{2}} = u^{1\frac{1}{2}}$ met $u = x^2 + 2$ dus $u' = 2x$

$g'(x) = 1\frac{1}{2}u^{\frac{1}{2}}\cdot u' = 1\frac{1}{2}(x^2+2)^{\frac{1}{2}}\cdot 2x = 3x\sqrt{x^2+2}$

c. $h(x) = \dfrac{3}{(2x^3+2)^5} = 3(2x^3+2)^{-5} = 3u^{-5}$ met $u = 2x^3 + 2$ dus $u' = 6x^2$

$h'(x) = -15u^{-6}\cdot u' = -15(2x^3+2)^{-6}\cdot 6x^2 = \dfrac{-90x^2}{(2x^3+2)^6}$

Opgave 8:

a. $f(x) = 2x^2(x^2 - 4x)^5$

$$f'(x) = 4x(x^2 - 4x)^5 + 2x^2 \cdot 5(x^2 - 4x)^4 \cdot (2x - 4) =$$
$$= 4x(x^2 - 4x)^5 + 10x^2(x^2 - 4x)^4(2x - 4)$$

b. $g(x) = (x^3 + x)\sqrt{x^3 + 2}$

$$g'(x) = (3x^2 + 1)\sqrt{x^3 + 2} + (x^3 + x) \cdot \frac{1}{2\sqrt{x^3 + 2}} \cdot 3x^2 =$$
$$= (3x^2 + 1)\sqrt{x^3 + 2} + \frac{3x^2(x^3 + x)}{2\sqrt{x^3 + 2}}$$

c. $h(x) = \dfrac{3x^2 + 6x}{(2x^3 + 2)^5}$

$$h'(x) = \frac{(2x^3 + 2)^5 \cdot (6x + 6) - (3x^2 + 6x) \cdot 5(2x^3 + 2)^4 \cdot 6x^2}{(2x^3 + 2)^{10}} =$$
$$= \frac{(2x^3 + 2)(6x + 6) - 30x^2(3x^2 + 6x)}{(2x^3 + 2)^6}$$
$$= \frac{12x^4 + 12x^3 + 12x + 12 - 90x^4 - 180x^3}{(2x^3 + 2)^6}$$
$$= \frac{-78x^4 - 168x^3 + 12x + 12}{(2x^3 + 2)^6}$$

Opgave 9:

a. $f'(x) = 1 \cdot \sqrt{50 - x^2} + x \cdot \dfrac{1}{2\sqrt{50 - x^2}} \cdot -2x = \sqrt{50 - x^2} - \dfrac{x^2}{\sqrt{50 - x^2}} =$

$$= \frac{50 - x^2}{\sqrt{50 - x^2}} - \frac{x^2}{\sqrt{50 - x^2}} = \frac{50 - 2x^2}{\sqrt{50 - x^2}} = 0$$

$50 - 2x^2 = 0$

$-2x^2 = -50$

$x^2 = 25$

$x = -5 \quad \vee \quad x = 5$

$(-5, -25) \quad (5, 25)$

b. $y_A = 7$

$f'(1) = 6\frac{6}{7}$

$y = 6\frac{6}{7}x + b$ door $(1,7)$

$7 = 6\frac{6}{7} + b$

$b = \frac{1}{7}$

$y = 6\frac{6}{7}x + \frac{1}{7}$

Opgave 10:

a. $f'(x)=\dfrac{(x^2+1)\cdot -1--x\cdot 2x}{(x^2+1)^2}=\dfrac{-x^2-1+2x^2}{(x^2+1)^2}=\dfrac{x^2-1}{(x^2+1)^2}=0$

$x^2-1=0$

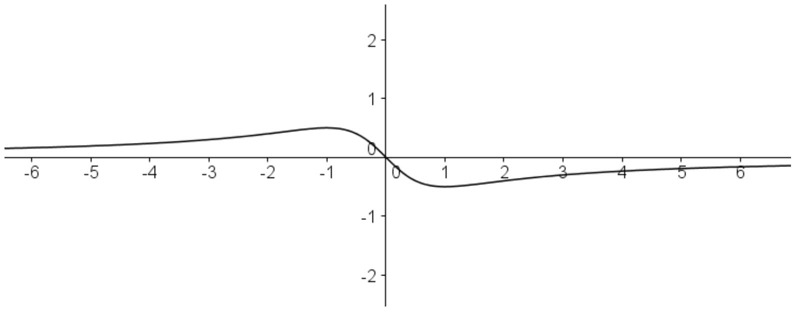
$x^2=1$

$x=-1 \quad \vee \quad x=1$

max $f(-1)=\frac{1}{2}$

min $f(1)=-\frac{1}{2}$

$B_f=[-\frac{1}{2},\frac{1}{2}]$



b. $f'(x)=\dfrac{x^2-1}{(x^2+1)^2}=\frac{3}{25}$

$3(x^2+1)^2=25(x^2-1)$

$3x^4+6x^2+3=25x^2-25$

$3x^4-19x^2+28=0$

$x^2=\dfrac{19\pm\sqrt{25}}{6}=\dfrac{19\pm 5}{6}$

$x^2=\frac{19+5}{6}=4 \quad \vee \quad x^2=\frac{19-5}{6}=2\frac{1}{3}$

$x=2 \quad \vee \quad x=-2 \quad \vee \quad x=\sqrt{2\frac{1}{3}} \quad \vee \quad x=-\sqrt{2\frac{1}{3}}$

Opgave 11:

$f_p'(x)=-x^2+2px+3=0$

$2px=x^2-3$

$p=\frac{1}{2}x-\dfrac{3}{2x}$ voor $x\neq 0$

$y=-\frac{1}{3}x^3+(\frac{1}{2}x-\dfrac{3}{2x})x^2+3x-4$

$y=-\frac{1}{3}x^3+\frac{1}{2}x^3-1\frac{1}{2}x+3x-4$

$y=\frac{1}{6}x^3+1\frac{1}{2}x-4$

Opgave 12:

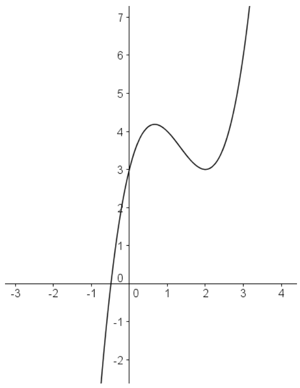
a. $f'(x)=3x^2-8x+4=0$

$x=\dfrac{8\pm\sqrt{16}}{6}=\dfrac{8\pm 4}{6}$

$x=\frac{8+4}{6}=2 \quad \vee \quad x=\frac{8-4}{6}=\frac{2}{3}$

max $f(\frac{2}{3})=4\frac{5}{27}$

min $f(2)=3$



b. $p<3 \quad \vee \quad p>4\frac{5}{27}$

c. $p=3 \quad \vee \quad p=4\frac{5}{27}$

d. $3<p<4\frac{5}{27}$

Opgave 13:

$$f'(x)=\frac{(x+1)(2x+2)-(x^2+2x+3)\cdot 1}{(x+1)^2}=\frac{2x^2+4x+2-x^2-2x-3}{(x+1)^2}=\frac{x^2+2x-1}{(x+1)^2}=\tfrac{1}{2}$$

$2(x^2+2x-1)=(x+1)^2$

$2x^2+4x-2=x^2+2x+1$

$x^2+2x-3=0$

$(x+3)(x-1)=0$

$x=-3 \quad \vee \quad x=1$

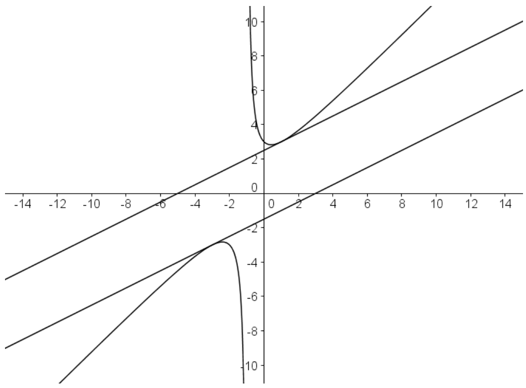
$y=-3 \qquad\quad y=3$

$y=\frac{1}{2}x+p$ door $(-3,-3)$ $\qquad y=\frac{1}{2}x+p$ door $(1,3)$

$-3=-1\frac{1}{2}+p \qquad\qquad 3=\frac{1}{2}+p$

$p=-1\frac{1}{2} \qquad\qquad\qquad p=2\frac{1}{2}$

dus $p\le -1\frac{1}{2} \quad \vee \quad p\ge 2\frac{1}{2}$

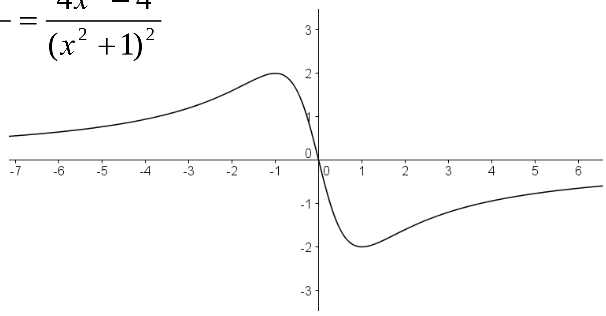


Opgave 14:

$$f'(x)=\frac{(x^2+1)\cdot -4--4x\cdot 2x}{(x^2+1)^2}=\frac{-4x^2-4+8x^2}{(x^2+1)^2}=\frac{4x^2-4}{(x^2+1)^2}$$

$f'(0)=-4$

$-4<a<0$



Opgave 15:

a. $f_p'(x)=-x^2+4x+p$

$f_p'(1)=-1+4+p=0$

$p=-3$

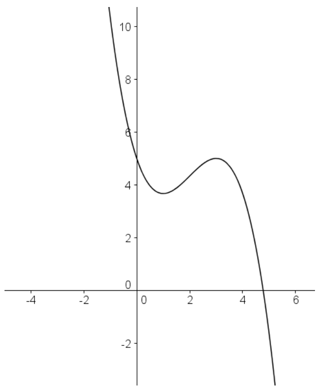
$f_{-3}'(x)=-x^2+4x-3=0$

$x^2-4x+3=0$

$(x-1)(x-3)=0$

$x=1 \quad \vee \quad x=3$

max $f_{-3}(3)=5$



b. $f_p'(x)=-x^2+4x+p$

$D=16+4p>0$

$4p>-16$

$p>-4$

Opgave 16:

$$f_p'(x)=\frac{(x+1)\cdot 2\cdot \dfrac{1}{2\sqrt{x}}-(2\sqrt{x}+p)\cdot 1}{(x+1)^2}=\frac{(x+1)\cdot \dfrac{1}{\sqrt{x}}-2\sqrt{x}-p}{(x+1)^2}$$

$$f_p'(4) = \frac{2\frac{1}{2} - 4 - p}{25} = -0{,}1$$
$$-1\tfrac{1}{2} - p = -2\tfrac{1}{2}$$
$$-p = -1$$
$$p = 1$$
$$f_1(x) = \frac{2\sqrt{x} + 1}{x + 1}$$
$$y_A = f_1(4) = 1$$
$y = -0{,}1x + q$ door $(4{,}1)$
$$1 = -0{,}4 + q$$
$$q = 1{,}4$$